HDV-003-1163002 Seat No. ________

M. Sc. (Mathematics) (Sem. III) (CBCS) Examination

November / December – 2017

MATH CMT - 3002 : Functional Analysis

Faculty Code : 003
Subject Code : 1163002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Answer all questions.
(2) Each question carries 14 marks.
(3) The figures to the right indicate marks allotted to the question.

1 All are compulsory : (Each question carries 2 marks) **14**

(a) True or false? Justify $\left(l^{\infty}, \|\cdot\|_{\infty}\right)$ has a Schauder basis.

(b) Define weak convergence, strong convergence in a n.l. Space.

(c) Define Banach Space.

(d) True or false? Justify Dual of a Hilbert space is a Hilbert space.

(e) Give an example of a space that is not Banach space over IK.

(f) True or false? Justify Every Separable Hilbert space is isomorphic to l^2.

(g) Define equivalent norms on a n.l. space.

2 Answer Any **Two** : **14**

(A) State and Prove the necessary and sufficient condition for a vector subspace of a Banach space to be a Banach space. True or false? Justify. $\left(C_0, \|\cdot\|_{\infty}\right)$ is a Banach space. 7

(B) State, without proof, Baire's theorem. Prove that a Banach space cannot have a countably infinite Hamel basis. **7**

(C) State and prove Riesz lemma. **7**

3 All are compulsory : **14**

(A) For a n.l. space X over IK, prove that the dual space X^I is separable $\Rightarrow X$ is separable. **7**

(B) Give an example to show that a metric on a vector space *x* need not be induced by a norm on *x*, with justification. **7**

OR

3 All are compulsory : **14**

(A) Let *X*, *Y* be a n.l. space over IK and $\|\cdot\|$ be the norm on $B\left(X.Y\right)$ defined by **7**

$\|T\| = \inf\{c > 0 / \|Tx\| \le c\|x\|, \forall\, x \in X\}$.

Prove that $\|T\| = \sup\left\{\frac{\|T_x\|}{\|x\|}, 0 \neq x \in X\right\} = \sup\{\|Tx\|, \|x\| = 1\}$.

(B) Define Canonical mapping C from a n.l. space X to X''. Prove that C : $X \to X''$ is an isometry. **7**

4 Answer any **two** : **14**

(A) State, without proof, projection theorem. If H is a Hilbert space and M is a non empty subset of H then prove that $\overline{span\ M} = M^{\perp\perp}$. **7**

(B) State and prove characterization of the Hyperspace in a n.l. Space. **7**

(C) State and prove closed graph theorem. **7**

5 All are compulsory : (Each question carries 2 marks) **14**

(A) State Hahn Banach Theorem.

(B) Define Hyper plane and Hyperspace and with an example.

(C) Give an example of a n.l. space which is not complete.

(D) If Y is closed subspace of a n.l. space X then give the definition of the induced norm on the quotient space X/Y.

(E) What is the meaning of the statement that the dual X' of X separates the points of X.

(F) Give the definitions of (1) Convergent series and (2) The absolute convergent series.

(G) Write the statement of Zorn's lemma and define sub linear functional.